NAG Toolbox for MATLAB

g01ff

1 Purpose

g01ff returns the deviate associated with the given lower tail probability of the gamma distribution, via the function name.

2 Syntax

```
[result, ifail] = g01ff(p, a, b, tol)
```

3 Description

The deviate, g_p, associated with the lower tail probability, p, of the gamma distribution with shape parameter α and scale parameter β, is defined as the solution to

$$P\left(G \leq g_p : \alpha, \beta\right) = p = \frac{1}{\beta^{\alpha}\Gamma(\alpha)}\int_0^{g_p} e^{-G/\beta}G^{\alpha-1}\,dG, \qquad 0 \leq g_p < \infty;\alpha, \beta > 0.$$

The method used is described by Best and Roberts 1975 making use of the relationship between the gamma distribution and the χ^2-distribution.

Let $y = 2\frac{g_p}{\beta}$. The required y is found from the Taylor series expansion

$$y = y_0 + \sum_r \frac{C_r(y_0)}{r!}\left(\frac{E}{\phi(y_0)}\right)^r,$$

where y_0 is a starting approximation

$$C_1(u) = 1,$$

$$C_{r+1}(u) = \left(r\Psi + \frac{d}{du}\right)C_r(u),$$

$$\Psi = \tfrac{1}{2} - \frac{\alpha - 1}{u},$$

$$E = p - \int_0^{y_0} \phi(u)\,du,$$

$$\phi(u) = \frac{1}{2^{\alpha}\Gamma(\alpha)}e^{-u/2}u^{\alpha-1}.$$

For most values of p and α the starting value

$$y_{01} = 2\alpha\left(z\sqrt{\frac{1}{9\alpha}} + 1 - \frac{1}{9\alpha}\right)^3$$

is used, where z is the deviate associated with a lower tail probability of p for the standard Normal distribution.

For p close to zero,

$$y_{02} = \left(p\alpha 2^{\alpha}\Gamma(\alpha)\right)^{1/\alpha}$$

is used.

For large p values, when $y_{01} > 4.4\alpha + 6.0$,

$$y_{03} = -2\left[\ln(1-p) - (\alpha - 1)\ln\left(\tfrac{1}{2}y_{01}\right) + \ln(\Gamma(\alpha))\right]$$

is found to be a better starting value than y_{01}.

For small α ($\alpha \leq 0.16$), p is expressed in terms of an approximation to the exponential integral and y_{04} is found by Newton–Raphson iterations.

Seven terms of the Taylor series are used to refine the starting approximation, repeating the process if necessary until the required accuracy is obtained.

4 References

Best D J and Roberts D E 1975 Algorithm AS91. The percentage points of the χ^2 distribution *Appl. Statist.* **24** 385–388

5 Parameters

5.1 Compulsory Input Parameters

1: **p – double scalar**

p, the probability from the required gamma distribution.

Constraint: $0.0 \leq \mathbf{p} < 1.0$.

2: **a – double scalar**

α, the shape parameter of the gamma distribution.

Constraint: $0.0 < \mathbf{a} \leq 10^6$.

3: **b – double scalar**

β, the scale parameter of the gamma distribution.

Constraint: $\mathbf{b} > 0$.

4: **tol – double scalar**

The relative accuracy required by you in the results. The smallest recommended value is $50 \times \delta$, where $\delta = \max\left(10^{-18}, \textit{machine precision}\right)$. If g01ff is entered with **tol** less than $10 \times \delta$ or greater or equal to 1.0, then $10 \times \delta$ is used instead.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

None.

5.4 Output Parameters

1: **result – double scalar**

The result of the function.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Note: g01ff may return useful information for one or more of the following detected errors or warnings.

If on exit **ifail** = 1, 2, 3 or 5, then g01ff returns 0.0.

ifail = 1

On entry, **p** < 0.0,
or **p** ≥ 1.0,

ifail = 2

On entry, **a** ≤ 0.0,
or **a** $> 10^6$,
or **b** ≤ 0.0

ifail = 3

p is too close to 0.0 or 1.0 to enable the result to be calculated.

ifail = 4

The solution has failed to converge in 100 iterations. A larger value of **tol** should be tried. The result may be a reasonable approximation.

ifail = 5

The series to calculate the gamma probabilities has failed to converge. This is an unlikely error exit.

7 Accuracy

In most cases the relative accuracy of the results should be as specified by **tol**. However, for very small values of α or very small values of p there may be some loss of accuracy.

8 Further Comments

None.

9 Example

```
p = 0.01;
a = 1;
b = 20;
tol = 0;
[result, ifail] = g01ff(p, a, b, tol)
```

```
result =
    0.2010
ifail =
           0
```